NONLOCAL SECOND-ORDER SHEAR DEFORMATION PLATE THEORY FOR FREE VIBRATION OF NANOPLATES

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Abstract

In this paper, the second-order shear deformation plate theory is developed for the study of the natural frequencies of rectangular nanoplates based on the nonlocal elasticity theory of Eringen. The governing equation of nanoplates is derived by using Hamilton's principle. The analytical solution for the natural frequencies and corresponding mode shapes of simply supported nanoplates is established. The effects of nonlocal parameters, the plate aspect ratios, and the plate thicknesses on the free vibration response are investigated. The obtained results show good agreement with other available solutions. The formulation and these analytical results of the proposed method could serve as a benchmark in the evaluation of future research.

Keywords: Nanoplates, free vibration, second-order shear deformation, nonlocal elasticity, simple support, analytical solution

Introduction

In recent years, nanostructures such as nanoplates have attracted worldwide attention from the research community for future application of nano-electromechanical systems (NEMS) since they have superior mechanical, chemical, and electronic properties. Nanoplates are widely used as resonators and sensors that may operate at very high frequencies up to 1 gigahertz. Therefore, a thorough understanding of the vibration behavior of nanoplates is important in the design of NEMS materials. It is well-known that, for the nanoscale structure, the classical continuum theory cannot predict the mechanical behavior of nanostructures. Thus, it is necessary to modify a continuum model to study the mechanical responses of nanostructures. The nonlocal elasticity has been widely developed for analyzing the mechanical behaviors of various nanostructures due to its reliable and accurate results. Moreover, this approach is less computationally expensive in comparison to molecular dynamics (MD). For the application of nonlocal elasticity to study the performance of nanoplates, Murmu and Pradhan (2009) investigated the influence of the small-scale effect on free in-plane vibration by employing a nonlocal continuum model. Aghababaei and Reddy (2009) studied the bending and vibration of a plate by using third-order shear deformation plate theory including the nonlocal effect.

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Ansari *et al*. (2010) investigated the free vibration response of single-layered graphene sheets with the nonlocal elasticity model and also compared their results with the MD method. Aksencer and Aydogdu (2011) analyzed the buckling and vibration of nanoplates using the Navier- and Levy-type solutions. Farajpour *et al*. (2011) investigated the buckling behavior of nanoplates of variable thicknesses under biaxial compression, and solved the problems by Galerkin's method. Satish *et al*. (2012) analyzed the thermal vibration of orthotropic nanoplates by using the 2-variable refined plate theory and nonlocal continuum mechanics. Wang and Li (2012) studied the bending behavior of a nanoplate embedded in an elastic matrix. Malekzadeh and Shojaee (2013) applied the 2-variable refined plate theory to study the free vibration of nanoplates. Pouresmaeeli *et al*. (2013) examined the vibration of viscoelastic orthotropic nanoplates embedded in a viscoelastic medium. Zenkour and Sobhy (2013) investigated the thermal buckling of nanoplates resting on a Winkler-Pasternak elastic medium, based on the sinusoidal shear deformation plate theory. Chakraverty and Behera (2014) studied the free vibration of rectangular nanoplates, and solved the problems by the Rayleigh–Ritz method. Recently, Panyatong *et al*. (2015) investigated the bending behavior of nanoplates embedded in an elastic medium including the nonlocal elasticity and surface stress. The classical plate theory (CPT) which neglects the effect of shear deformations can induce inaccurate results for the analysis of thick plates. The first-order shear deformation theory (FSDT) accounts for the transverse shear deformation by assuming a constant shear strain throughout the plate thickness and requires a shear correction factor in order to satisfy the zero transverse shear stress at the top and bottom of the plates. To avoid using a shear correction factor but retaining consideration of the transverse shear strain and rotation, the second-order shear deformation theory (SSDT) is used for this work because the formulation of the displacement field has a simple form.

From a literature review, the free vibration of homogenous nanoplates by using the SSDT has never been formulated. Thus, the main purpose of this work is to study the free vibration behavior of nanoplates by developing the SSDT in conjunction with the nonlocal elasticity. Comparisons of the obtained results with those of other available solutions are performed to verify the reliability of the present formulation. Finally, the influences of nonlocal parameters, plate aspect ratios, and thicknesses on the free vibration responses are studied.

Formulation of the Problem

A rectangular nanoplate of thickness h, length a, and width b is considered with the Cartesian coordinate system (x, y, z), as shown in Figure 1. The SSDT with nonlocal elasticity is developed to derive the governing equations for the vibration problem. The displacements at any

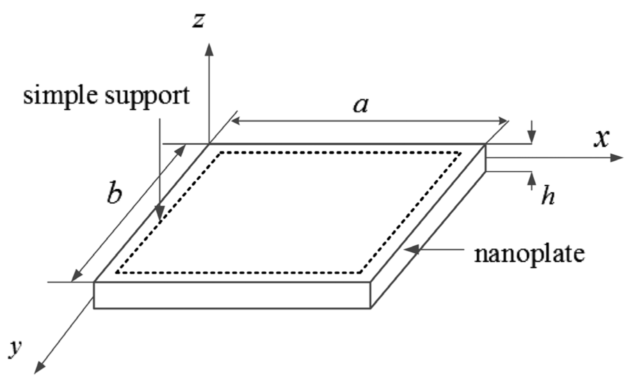

Figure 1. Geometric of a uniform rectangular nanoplate

material point depend only on the middle plane of the nanoplate and the straight line normal to the middle plane is defined by a quadratic curve after deformation. Although the surface energy has a significant influence on the response of nanostructures, the presented formulation is confined by not including the surface energy effect.

Nonlocal Elasticity Theory

The nonlocal elasticity theory was first proposed by Eringen (1983, 2002). The nonlocal elasticity theory is based on the crucial concept that the stress at a point is a function of the strain at all points of the body. According to the nonlocal elasticity theory, the constitutive relations can be represented by the following differential equations as

$$\tilde{L}\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\\ \sigma_{xz}\\ \sigma_{yz}\end{Bmatrix}=\begin{bmatrix}q_{11} & q_{12} & 0 & 0 & 0\\ q_{12} & q_{22} & 0 & 0 & 0\\ 0 & 0 & q_{33} & 0 & 0\\ 0 & 0 & 0 & q_{44} & 0\\ 0 & 0 & 0 & 0 & q_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{xy}\\ \varepsilon_{xz}\\ \varepsilon_{yz}\end{Bmatrix}, \quad (1)$$

where $\tilde{L}=\left(1-\mu\nabla^2\right)$, $q_{11}=q_{22}=E/(1-\nu^2)$, $q_{12}=\nu E/\left(1-\nu^2\right)$, and $q_{33}=q_{44}=q_{55}=2G$. Moreover E, G, and v are the modulus of elasticity, the shear modulus, and Poisson's ratio, respectively. The scale factor $\mu=\left(e_0 l_i\right)^2$ is the nonlocal parameter, where l_i is an internal characteristic length (such as lattice spacing, granular distance, and distance between C-C bonds) and e_0 is a material constant which is determined to calibrate the nonlocal model with the experimental results or the results of MD simulations.

The Second-Order Shear Deformation Plate Theory and the Governing Equation of Motion

According to the SSDT, the displacement field can be expressed as

$$u=u_0(x,y,t)+z\phi_1(x,y,t)+z^2\phi_2(x,y,t), \quad (2a)$$

$$v=v_0(x,y,t)+z\psi_1(x,y,t)+z^2\psi_2(x,y,t), \quad (2b)$$

$$w=w_0(x,y,t), \quad (2c)$$

where u_0, v_0 and w_0 are the displacement components of the material point at the middle plane of the plate; $\emptyset_1$ and ψ_1 are the rotations for the y and x axes, respectively; and $\emptyset_2$ and ψ_2 are variables of the second-order terms. Furthermore, the strain-displacement relations can be expressed as

$$\varepsilon_{xx}=\frac{\partial u_0}{\partial x}+z\frac{\partial\phi_1}{\partial x}+z^2\frac{\partial\phi_2}{\partial x}, \quad (3a)$$

$$\varepsilon_{yy}=\frac{\partial v_0}{\partial y}+z\frac{\partial\psi_1}{\partial y}+z^2\frac{\partial\psi_2}{\partial y}, \quad (3b)$$

$$\varepsilon_{xy}=\frac{1}{2}\left\{\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}+z\left(\frac{\partial\phi_1}{\partial y}+\frac{\partial\psi_1}{\partial x}\right)+z^2\left(\frac{\partial\phi_2}{\partial y}+\frac{\partial\psi_2}{\partial x}\right)\right\}, \quad (3c)$$

$$\varepsilon_{xz}=\frac{1}{2}\left(\phi_1+2z\phi_2+\frac{\partial w}{\partial x}\right), \quad (3d)$$

$$\varepsilon_{yz}=\frac{1}{2}\left(\psi_1+2z\psi_2+\frac{\partial w}{\partial y}\right). \quad (3e)$$

Hamilton's principle is employed to derive the equations of motion of nanoplates. Referring to Hamilton's principle, the following equation is obtained:

$$\int_0(\delta K-\delta U)dt=0, \quad (4)$$

where δK, δU, and T are the variation of the kinetic energy, the strain energy, and the final time, respectively. The variation of the kinetic energy is given by

$$\delta K=\int_{\Omega}\int_{-h/2}^{h/2}\rho\left(\ddot{u}\delta u+\ddot{v}\delta v+\ddot{w}\delta w\right)dzd\Omega, \quad (5)$$

where ρ is the mass density of nanoplates and Ω is the area of the middle plane of nanoplates. The variation of the strain energy is given by

$$\delta U=\int_V\left(\sigma_{xx}\delta\varepsilon_{xx}+\sigma_{yy}\delta\varepsilon_{yy}+2\sigma_{xy}\delta\varepsilon_{xy}+2\sigma_{xz}\delta\varepsilon_{xz}+2\sigma_{yz}\delta\varepsilon_{yz}\right)dV. \quad (6)$$

where V is the volume of nanoplates. Substituting Equations (5) and (6) into Equation (4) and then using Equations (1)-(3), we can derive the equations of motion, as follows:

$$\delta u_0:\quad \frac{\partial N_{xx}}{\partial x}+\frac{\partial N_{xy}}{\partial y}=I_0\ddot{u}+I_2\ddot{\phi}_2, \quad (7a)$$

$$\delta v_0: \quad \frac{\partial N_{yy}}{\partial y} + \frac{\partial N_{xy}}{\partial x} = I_0 \ddot{v} + I_2 \ddot{\psi}_2, \tag{7b}$$

$$\delta w_0: \quad \frac{\partial Q_{xz}}{\partial x} + \frac{\partial Q_{yz}}{\partial y} = I_0 \ddot{w}, \tag{7c}$$

$$\delta \phi_1: \quad \frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_{xz} = I_2 \ddot{\phi}_1, \tag{7d}$$

$$\delta \phi_2: \quad \frac{\partial L_{xx}}{\partial x} + \frac{\partial L_{xy}}{\partial y} - 2R_{xz} = I_2 \ddot{u} + I_4 \ddot{\phi}_2, \tag{7e}$$

$$\delta \psi_1: \quad \frac{\partial M_{yy}}{\partial x} + \frac{\partial M_{xy}}{\partial x} - Q_{yz} = I_2 \ddot{\psi}_1, \tag{7f}$$

$$\delta \psi_2: \quad \frac{\partial L_{yy}}{\partial x} + \frac{\partial L_{xy}}{\partial x} - 2R_{yz} = I_2 \ddot{v} + I_4 \ddot{\psi}_2, \tag{7g}$$

where N, M, L, Q and R are the nonlocal stress resultants and I_0, I_2, and I_4 are the mass inertias which are defined as

$$\tilde{L}N_{xx} = \left\{ h\left(q_{11}\frac{\partial u}{\partial x} + q_{12}\frac{\partial v}{\partial y} \right) + \frac{h^3}{12}\left(q_{11}\frac{\partial \phi_2}{\partial x} + q_{12}\frac{\partial \psi_2}{\partial y} \right) \right\}, \tag{8a}$$

$$\tilde{L}N_{yy} = \left\{ h\left(q_{12}\frac{\partial u}{\partial x} + q_{22}\frac{\partial v}{\partial y} \right) + \frac{h^3}{12}\left(q_{12}\frac{\partial \phi_2}{\partial x} + q_{22}\frac{\partial \psi_2}{\partial y} \right) \right\}, \tag{8b}$$

$$\tilde{L}N_{xy} = \left\{ hq_{33}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \frac{h^3}{12}q_{33}\left(\frac{\partial \phi_2}{\partial y} + \frac{\partial \psi_2}{\partial x} \right) \right\}, \tag{8c}$$

$$\tilde{L}M_{xx} = \frac{h^3}{12}\left(q_{11}\frac{\partial \phi_1}{\partial x} + q_{12}\frac{\partial \psi_1}{\partial y} \right), \tag{8d}$$

$$\tilde{L}M_{yy} = \frac{h^3}{12}\left(q_{12}\frac{\partial \phi_1}{\partial x} + q_{22}\frac{\partial \psi_1}{\partial y} \right), \tag{8e}$$

$$\tilde{L}M_{xy} = \frac{h^3}{12}q_{33}\left(\frac{\partial \phi_1}{\partial y} + \frac{\partial \psi_1}{\partial x} \right), \tag{8f}$$

$$\tilde{L}L_{xx} = \left\{ \frac{h^3}{12}\left(q_{11}\frac{\partial u}{\partial x} + q_{12}\frac{\partial v}{\partial y} \right) + \frac{h^5}{80}\left(q_{11}\frac{\partial \phi_2}{\partial x} + q_{12}\frac{\partial \psi_2}{\partial y} \right) \right\}, \tag{8g}$$

$$\tilde{L}L_{yy} = \left\{ \frac{h^3}{12}\left(q_{12}\frac{\partial u}{\partial x} + q_{22}\frac{\partial v}{\partial y} \right) + \frac{h^5}{80}\left(q_{12}\frac{\partial \phi_2}{\partial x} + q_{22}\frac{\partial \psi_2}{\partial y} \right) \right\}, \tag{8h}$$

$$\tilde{L}L_{xy} = \left\{ \frac{h^3}{12}q_{33}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \frac{h^5}{80}q_{11}\left(\frac{\partial \phi_2}{\partial y} + \frac{\partial \psi_2}{\partial x} \right) \right\}, \tag{8i}$$

$$\tilde{L}Q_{xz} = hq_{44}\left(\phi_1 + \frac{\partial w}{\partial x} \right), \tag{8j}$$

$$\tilde{L}Q_{yz} = hq_{55}\left(\psi_1 + \frac{\partial w}{\partial y} \right), \tag{8k}$$

$$\tilde{L}R_{xz} = \frac{h^3}{6}q_{44}\phi_2, \tag{8l}$$

$$\tilde{L}R_{yz} = \frac{h^3}{6}q_{55}\psi_2, \tag{8m}$$

$$I_0 = \rho h, \quad I_2 = \frac{\rho h^3}{12}, \quad I_4 = \frac{\rho h^5}{80}. \tag{8n)-(8p}$$

Finally, substituting Equations (8a)-(8m) into Equations (7a)-(7g), we obtain the governing equations of motion in terms of the displacement field, as follows:

$$\left\{ \begin{array}{l} h\left(q_{11}\frac{\partial^2 u}{\partial x^2} + q_{33}\frac{\partial^2 u}{\partial y^2} \right) + h(q_{12}+q_{33})\frac{\partial^2 v}{\partial x \partial y} \\ + \frac{h^3}{12}\left(q_{11}\frac{\partial^2 \phi_2}{\partial x^2} + q_{33}\frac{\partial^2 \phi_2}{\partial y^2} \right) + \frac{h^3}{12}(q_{12}+q_{33})\frac{\partial^2 \psi_2}{\partial x \partial y} \end{array} \right\} = \tilde{L}\left(I_0\ddot{u} + I_2\ddot{\phi}_2 \right), \tag{9a}$$

$$\left\{ \begin{array}{l} h(q_{12}+q_{33})\frac{\partial^2 u}{\partial x \partial y} + h\left(q_{33}\frac{\partial^2 v}{\partial x^2} + q_{22}\frac{\partial^2 v}{\partial y^2} \right) \\ + \frac{h^3}{12}(q_{12}+q_{33})\frac{\partial^2 \phi_2}{\partial x \partial y} + \frac{h^3}{12}\left(q_{33}\frac{\partial^2 \psi_2}{\partial x^2} + q_{22}\frac{\partial^2 \psi_2}{\partial y^2} \right) \end{array} \right\} = \tilde{L}\left(I_0\ddot{v} + I_2\ddot{\psi}_2 \right), \tag{9b}$$

$$h\left\{ q_{44}\frac{\partial^2 w}{\partial x^2} + q_{55}\frac{\partial^2 w}{\partial y^2} + q_{44}\frac{\partial \phi_1}{\partial x} + q_{55}\frac{\partial \psi_1}{\partial y} \right\} = \tilde{L}I_0\ddot{w}, \tag{9c}$$

$$\left\{ \frac{h^3}{12}\left(q_{11}\frac{\partial^2 \phi_1}{\partial x^2} + q_{44}\frac{\partial^2 \phi_1}{\partial y^2} \right) + \frac{h^3}{12}(q_{12}+q_{44})\frac{\partial^2 \psi_1}{\partial x \partial y} - hq_{55}\left(\frac{\partial w}{\partial x} + \phi_1 \right) \right\} = \tilde{L}I_2\ddot{\phi}_1, \tag{9d}$$

$$\left\{ \begin{array}{l} \frac{h^3}{12}\left(q_{11}\frac{\partial^2 u}{\partial x^2} + q_{33}\frac{\partial^2 u}{\partial y^2} \right) + \frac{h^3}{12}(q_{12}+q_{33})\frac{\partial^2 v}{\partial x \partial y} \\ + \frac{h^5}{80}\left(q_{11}\frac{\partial^2 \phi_2}{\partial x^2} + q_{33}\frac{\partial^2 \phi_2}{\partial y^2} \right) + \frac{h^5}{80}(q_{12}+q_{33})\frac{\partial^2 \psi_2}{\partial x \partial y} - \frac{h^3}{3}q_{44}\phi_2 \end{array} \right\} = \tilde{L}\left(I_2\ddot{u} + I_4\ddot{\phi}_2 \right), \tag{9e}$$

$$\left\{ \begin{array}{l} \frac{h^3}{12}\left(q_{33}\frac{\partial^2 \psi_1}{\partial x^2} + q_{22}\frac{\partial^2 \psi_1}{\partial y^2} \right) \\ + \frac{h^3}{12}(q_{12}+q_{33})\frac{\partial^2 \phi_1}{\partial x \partial y} - hq_{55}\left(\frac{\partial w}{\partial y} + \psi_1 \right) \end{array} \right\} = \tilde{L}I_2\ddot{\psi}_1, \tag{9f}$$

$$\left\{ \begin{array}{l} \frac{h^3}{12}\left(q_{33}\frac{\partial^2 v}{\partial x^2} + q_{22}\frac{\partial^2 v}{\partial y^2} \right) + \frac{h^3}{12}(q_{12}+q_{33})\frac{\partial^2 u}{\partial x \partial y} \\ + \frac{h^5}{80}\left(q_{33}\frac{\partial^2 \psi_2}{\partial x^2} + q_{22}\frac{\partial^2 \psi_2}{\partial y^2} \right) + \frac{h^5}{80}(q_{12}+q_{33})\frac{\partial^2 \phi_2}{\partial x \partial y} - \frac{h^3}{3}q_{55}\psi_2 \end{array} \right\} \tag{9g}$$

$$= \tilde{L}\left(I_2\ddot{v} + I_4\psi_2 \right).$$

Note that the equations of motion of the first-order shear deformation theory can be obtained from the above equations by neglecting the variables $\varnothing_2$ and ψ_2.

Solution of the problem

In this section, the governing differential equations for the free vibration of the nanoplates including the nonlocal effects have been solved by Navier's approach for the simply supported boundary conditions. The simply supported boundary conditions for a rectangular plate are

a) $v = 0, w = 0, \psi_1 = 0, \psi_2 = 0, N_{xx} = 0,$ (10a)
$M_{xx} = 0$ and $L_{xy} = 0$ at $x = 0, a,$

b) $u = 0, w = 0, \emptyset_1 = 0, \emptyset_2 = 0, N_{yy} = 0,$ (10b)
$M_{yy} = 0$ and $L_{xy} = 0$ at $y = 0, b.$

The displacement solution can be expressed as

$$u = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} U_{mn} e^{-i\omega_{mn}t} \cos\alpha x \sin\beta y, \tag{11a}$$

$$v = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} V_{mn} e^{-i\omega_{mn}t} \sin\alpha x \cos\beta y, \tag{11b}$$

$$w = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} W_{mn} e^{-i\omega_{mn}t} \sin\alpha x \sin\beta y, \tag{11c}$$

$$\phi_1 = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \Phi_{1mn} e^{-i\omega_{mn}t} \cos\alpha x \sin\beta y, \tag{11d}$$

$$\phi_2 = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \Phi_{2mn} e^{-i\omega_{mn}t} \cos\alpha x \sin\beta y, \tag{11e}$$

$$\psi_1 = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \Psi_{1mn} e^{-i\omega_{mn}t} \sin\alpha x \cos\beta y, \tag{11f}$$

$$\psi_2 = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \Psi_{2mn} e^{-i\omega_{mn}t} \sin\alpha x \cos\beta y, \tag{11g}$$

where $i = \sqrt{-1}$, $\alpha = m\pi / a$, and $\beta = n\pi / b$, and m and n denote the half wave numbers for the x and y directions, respectively. By using Equations (11a)-(11g), the simply supported boundary conditions for the nanoplates, i.e. Equations (10a) and (10b), will be satisfied automatically. Substituting Equations (11a)-(11g) into Equations (9a)-(9g), we obtain the system of the linear equations as

$$\left([C]_{7x7} - \omega_{mn}^2 [M]_{7x7}\right) \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ \Phi_{1mn} \\ \Phi_{2mn} \\ \Psi_{1mn} \\ \Psi_{2mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}, \tag{12}$$

where the matrix $[C]_{7x7}$ and $[M]_{7x7}$ are given in the Appendix. Equation (12) is a standard eigenvalue problem in which the eigenvalues are found by setting the determinant of $\left([C] - \omega_{mn}^2 [M]\right)$ to zero. The obtained eigenvalues are the natural frequencies of nanoplates. For the presentation of the analytical results, the following dimensionless natural frequency is introduced:

$$\bar{\omega}_{mn} = \omega_{mn} h \sqrt{\rho / G}. \tag{13}$$

Numerical Results and Discussion

In this section, the numerical results are presented to study the free vibration behavior of the nanoplates with simply supported boundary conditions.

Reliability

In order to present the reliability of the proposed analytical solution, the obtained results are compared with those of other available plate theories in open literature with the material properties of the nanoplates, as listed in Table 1. In Table 2, the dimensionless natural frequencies for the different values of nonlocal parameters are contained. It can be seen that, in all cases, the present results are in good agreement with the classical, first-order, third-order, and 2-variable refined plate theories. Especially, they are close to the results calculated from the third-order shear deformation plate theory. Moreover, the proposed formulation is employed to analyze single-layered grapheme sheets (SLGS). The material properties of SLGS are contained in Table 3. The analytical results of zigzag and armchair SLGS are shown in Tables 4 and 5, respectively, and are also compared with those from MD. It is clearly observed that the present results are very close to the results of MD. Thus, the proposed formulation has accuracy and reliability for the prediction of the free vibration

Table 1. Material properties of nanoplates

Properties Aghababaei and Reddy (2009)	
Modulus of elasticity, E	30×10^6 Pa
Poisson's ratio, v	0.3
Mass density, ρ	1.0 N/a^3

response of nanoplates.

Parametric Studies

The dimensionless natural frequencies as nonlocal parameters for the different plate theories are plotted in Figures 2(a)-(c). It is clearly observed that the nonlocal parameters significantly affect the vibration behavior of nanoplates. The dimensionless natural frequency decreases for all mode shapes with increasing the nonlocal parameter μ. Thus, the local nanoplate model with ($\mu = 0$ mm^2) overestimates the free vibration response of nanoplates. To evaluate the influence of the plate aspect ratios on the free vibration of nanoplates, Figures 3(a)-(c) are established. It is evidently observed that the dimensionless natural frequency increases when the length-to-width

Table 2. The dimensionless natural frequency $\bar{\omega}_{11}$ of simply supported nanaoplates (a = 10 nm)

b/a	a/h	μ	Aghababaei and Reddy (2009)			Malekzadeh and Shojaee (2013)	Present
			Classical	First-order	Third-order	2- variable	
1	10	0	0.0963	0.0930	0.0935	0.0930	0.0934
		1	0.0880	0.0850	0.0854	0.0850	0.0854
		2	0.0816	0.0788	0.0791	0.0788	0.0791
		3	0.0763	0.0737	0.0741	0.0737	0.0740
		4	0.0720	0.0696	0.0699	-	0.0698
		5	0.0683	0.0660	0.0663	-	0.0663
	20	0	0.0241	0.0239	0.0239	0.0239	0.0239
		1	0.0220	0.0218	0.0218	0.0218	0.0218
		2	0.0204	0.0202	0.0202	0.0202	0.0202
		3	0.0191	0.0189	0.0189	0.0189	0.0189
		4	0.0180	0.0178	0.0179	-	0.0179
		5	0.0171	0.0169	0.0170	-	0.0169
2	10	0	0.0602	0.0589	0.0591	0.0589	0.0590
		1	0.0568	0.0556	0.0557	0.0556	0.0557
		2	0.0539	0.0527	0.0529	0.0527	0.0529
		3	0.0514	0.0503	0.0505	0.0503	0.0504
		4	0.0493	0.0482	0.0483	-	0.0483
		5	0.0473	0.0463	0.0464	-	0.0464
	20	0	0.0150	0.0150	0.0150	0.0150	0.0150
		1	0.0142	0.0141	0.0141	0.0141	0.0141
		2	0.0135	0.0134	0.0134	0.0134	0.0134
		3	0.0129	0.0128	0.0128	0.0128	0.0128
		4	0.0123	0.0123	0.0123	-	0.0123
		5	0.0118	0.0118	0.0118	-	0.0118

ratio (a / b) increases, because the higher values of the length-to-width ratio have more stiffness than the lower ones. For high values of the length-to-width ratio, there are large differences in the dimensionless natural frequency between including and excluding the shear deformation effect. Finally, to study the effects of the nanoplates' thickness on the vibration characteristic, Figures 4(a)-(c) are plotted. It is

Table 3. Material properties of graphene sheet

Properties Ansari *et al.* (2010)	
Modulus of elasticity, E	1×10^{12} Pa
Poisson's ratio, ν	0.16
Mass density, ρ	2250 kg/m^3
Thickness, h	0.34 nm

Table 4. The natural frequency of simply-supported zigzag single-layered graphene sheets (SLGS)

Length of square SLGS, (nm)	Ansari *et al.* (2010) (1) Molecular dynamics, (THz)	Ansari *et al.* (2010) (2) First-order, μ = 1.41 nm^2 (THz)	(3) Present, μ = 1.41 nm^2 (THz)	Difference between (1) and (2), (%)	Difference between (1) and (3), (%)
10	0.0587725	0.0584221	0.0580588	0.60	1.21
15	0.0273881	0.0282888	0.0275709	3.29	0.67
20	0.0157524	0.0164593	0.0159061	4.49	0.98
25	0.0099840	0.0107085	0.0103042	7.26	3.21
30	0.0070655	0.0075049	0.0072039	6.22	1.96
35	0.0052982	0.0055447	0.0053143	4.65	0.30
40	0.0040985	0.0042608	0.0040797	3.96	0.46
45	0.0032609	0.0033751	0.0032294	3.50	0.97
50	0.0026194	0.0027388	0.0026193	4.56	0.01

Table 5. The natural frequency of simply-supported armchair single-layered graphene sheets (SLGS)

Length of square SLGS, (nm)	Ansari *et al.* (2010) (1) Molecular dynamics, (THz)	Ansari *et al.* (2010) (2) First-order, μ = 1.34 nm^2 (THz)	(3) Present, μ = 1.34 nm^2 (THz)	Difference between (1) and (2), (%)	Difference between (1) and (3), (%)
10	0.0595014	0.0592359	0.058375	0.45	1.89
15	0.0277928	0.0284945	0.027647	2.52	0.53
20	0.0158141	0.0165309	0.015932	4.53	0.74
25	0.0099975	0.0107393	0.010315	7.42	3.18
30	0.0070712	0.0075201	0.007209	6.35	1.95
35	0.0052993	0.0055531	0.005317	4.79	0.34
40	0.0041017	0.0042657	0.004081	4.00	0.49
45	0.0032614	0.0033782	0.003231	3.58	0.95
50	0.0026197	0.0027408	0.002620	4.62	0.01

evident that the influence of the increases in the nanoplates' thickness leads to the dimensionless natural frequency increases for all nonlocal parameters. This also means that increasing the thickness increases the stiffness of the nanoplates.

Conclusions

In this study, the linear free vibration of nanoplates is studied based on the SSDT and the nonlocal elasticity theory. The governing equation of the nanoplate model is derived. The solution for the free vibration of simply supported nanoplates is established. From the numerical study, it is obvious that the small-scale effect from the nonlocal elasticity decreases the natural frequency of nanoplates. The analytical formulation of the proposed method could serve as a benchmark in the evaluation of future research.

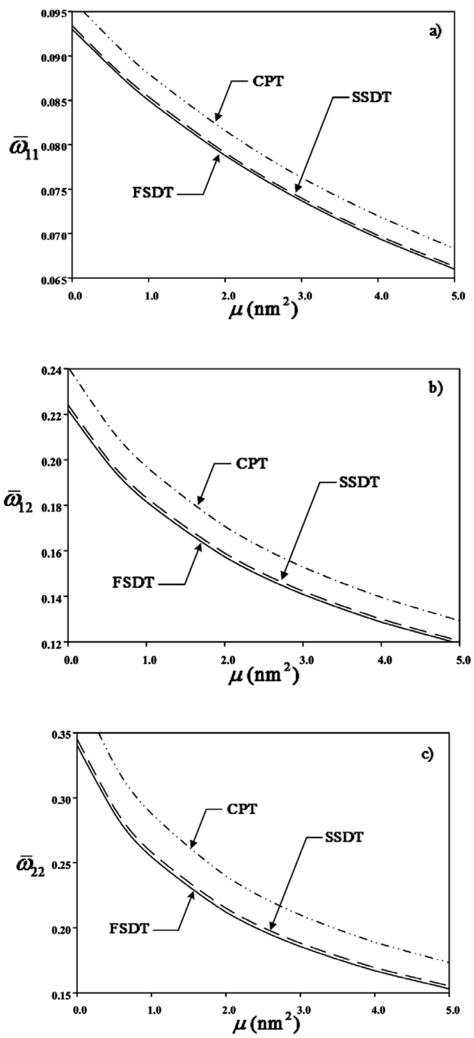

Figure 2. **(a)-(c) The dimensionless natural frequencies as nonlocal parameters with different plate theories for a/h = 10. (CPT = classical plate theory, FSDT = first-order shear deformation theory, SSDT = second-order shear deformation theory)**

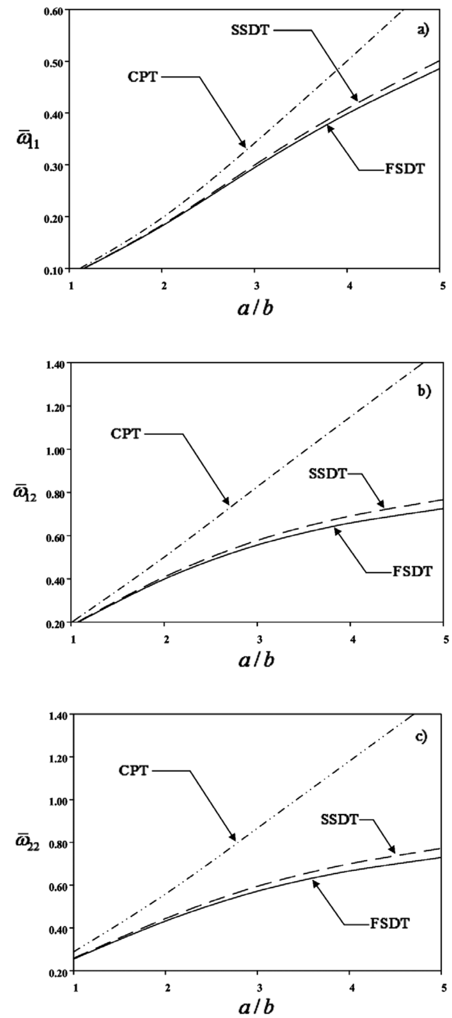

Figure 3. **(a)-(c). The dimensionless natural frequencies as the length-to-width ratio of nonlocal parameters with different plate theories for a/h = 10 and μ = 1 nm^2. (CPT = classical plate theory, FSDT = first-order shear deformation theory, SSDT = second-order shear deformation theory)**

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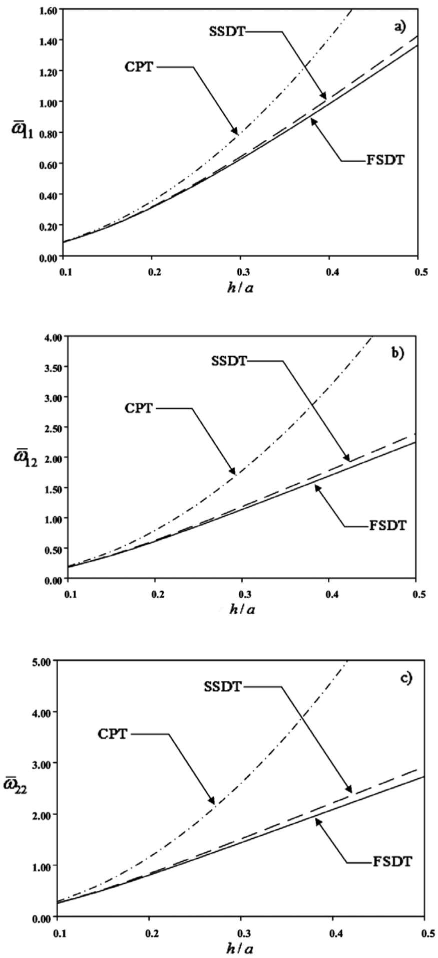

Figure 4 **(a)-(c). The dimensionless natural frequencies as the thickness-to-length ratio of nonlocal parameters with different plate theories for a/b = 1 and μ = 1 nm². (CPT = classical plate theory, FSDT = first-order shear deformation theory, SSDT = second-order shear deformation theory)**

Appendix

Coefficients C_{11} through C_{77} and M_{11} through M_{77} :

$$C_{11} = h(q_{11}\alpha^2 + q_{33}\beta^2)\,, \tag{A1}$$
$$C_{22} = h(q_{33}\alpha^2 + q_{22}\beta^2)\,, \tag{A2}$$
$$C_{33} = h(q_{44}\alpha^2 + q_{33}\beta^2)\,, \tag{A3}$$
$$C_{44} = h^3(q_{11}\alpha^2 + q_{33}\beta^2)/12 + hq_{33}\,, \tag{A4}$$
$$C_{55} = h^5(q_{11}\alpha^2 + q_{33}\beta^2)/80 + h^3 q_{44}/3, \tag{A5}$$
$$C_{66} = h^3(q_{33}\alpha^2 + q_{22}\beta^2)/12 + hq_{33}\,, \tag{A6}$$
$$C_{77} = h^5(q_{33}\alpha^2 + q_{22}\beta^2)/80 + h^3 q_{33}/3\,, \tag{A7}$$
$$C_{12} = C_{21} = h(q_{12} + q_{33})\alpha\beta\,, \tag{A8}$$
$$C_{13} = C_{31} = 0\,, \tag{A9}$$
$$C_{14} = C_{41} = 0\,, \tag{A10}$$
$$C_{15} = C_{51} = h^3(q_{11}\alpha^2 + q_{33}\beta^2)/12\,, \tag{A11}$$
$$C_{16} = C_{61} = 0\,, \tag{A12}$$
$$C_{17} = C_{71} = h^3(q_{12} + q_{33})\alpha\beta/12\,, \tag{A13}$$
$$C_{23} = C_{32} = 0\,, \tag{A14}$$
$$C_{24} = C_{42} = 0\,, \tag{A15}$$
$$C_{25} = C_{52} = h^3(q_{12} + q_{33})\alpha\beta/12\,, \tag{A16}$$
$$C_{26} = C_{62} = 0\,, \tag{A17}$$
$$C_{27} = C_{72} = h^3(q_{33}\alpha^2 + q_{22}\beta^2)/12\,, \tag{A18}$$
$$C_{34} = C_{43} = hq_{33}\alpha\,, \tag{A19}$$
$$C_{35} = C_{53} = 0\,, \tag{A20}$$
$$C_{36} = C_{63} = hq_{55}\beta\,, \tag{A21}$$
$$C_{37} = C_{73} = 0\,, \tag{A22}$$
$$C_{45} = C_{54} = 0\,, \tag{A23}$$
$$C_{46} = C_{64} = h^3(q_{12} + q_{33})\alpha\beta/12\,, \tag{A24}$$
$$C_{47} = C_{74} = 0\,, \tag{A25}$$
$$C_{56} = C_{65} = 0\,, \tag{A26}$$
$$C_{57} = C_{75} = h^5(q_{12} + q_{33})\alpha\beta/80\,, \tag{A27}$$
$$C_{67} = C_{76} = 0\,, \tag{A28}$$
$$M_{11} = I_0\left(1 + \mu\left(\alpha^2 + \beta^2\right)\right), \tag{A29}$$
$$M_{22} = I_0\left(1 + \mu\left(\alpha^2 + \beta^2\right)\right), \tag{A30}$$
$$M_{33} = I_0\left(1 + \mu\left(\alpha^2 + \beta^2\right)\right), \tag{A31}$$
$$M_{44} = I_2\left(1 + \mu\left(\alpha^2 + \beta^2\right)\right), \tag{A32}$$
$$M_{55} = I_4\left(1 + \mu\left(\alpha^2 + \beta^2\right)\right), \tag{A33}$$

$$M_{66} = I_2\left(1+\mu\left(\alpha^2+\beta^2\right)\right), \quad \text{(A34)}$$

$$M_{77} = I_4\left(1+\mu\left(\alpha^2+\beta^2\right)\right), \quad \text{(A35)}$$

$$M_{12} = M_{21} = 0, \quad \text{(A36)}$$

$$M_{13} = M_{31} = 0, \quad \text{(A37)}$$

$$M_{14} = M_{41} = 0, \quad \text{(A38)}$$

$$M_{15} = M_{51} = I_2\left(1+\mu\left(\alpha^2+\beta^2\right)\right), \quad \text{(A39)}$$

$$M_{16} = M_{61} = 0, \quad \text{(A40)}$$

$$M_{17} = M_{71} = 0, \quad \text{(A41)}$$

$$M_{23} = M_{32} = 0, \quad \text{(A42)}$$

$$M_{24} = M_{42} = 0, \quad \text{(A43)}$$

$$M_{25} = M_{52} = 0, \quad \text{(A44)}$$

$$M_{26} = M_{62} = 0, \quad \text{(A45)}$$

$$M_{27} = M_{72} = I_2\left(1+\mu\left(\alpha^2+\beta^2\right)\right), \quad \text{(A46)}$$

$$M_{34} = M_{43} = 0, \quad \text{(A47)}$$

$$M_{35} = M_{53} = 0, \quad \text{(A48)}$$

$$M_{36} = M_{63} = 0, \quad \text{(A49)}$$

$$M_{37} = M_{73} = 0, \quad \text{(A50)}$$

$$M_{45} = M_{54} = 0, \quad \text{(A51)}$$

$$M_{46} = M_{64} = 0, \quad \text{(A52)}$$

$$M_{47} = M_{74} = 0, \quad \text{(A53)}$$

$$M_{56} = M_{65} = 0, \quad \text{(A54)}$$

$$M_{57} = M_{75} = 0, \quad \text{(A55)}$$

$$M_{67} = M_{76} = 0, \quad \text{(A56)}$$

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